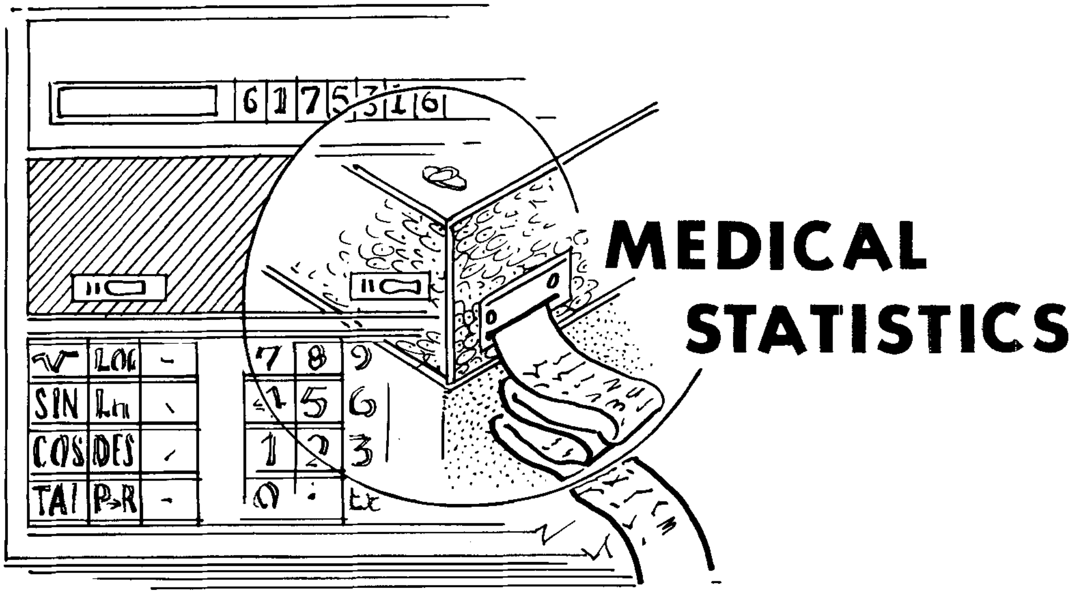

Statistical Pearls III. The Most Likely Interpretation

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PROBABILITY is probably the most important word in modern biomedical research. Yet, it would be difficult for most of us to answer the question, "What is probability?" In a sense, probability is like a pregnant woman in that it is a currently existing condition which defines an *expected* future event. Probability is a prediction of the *future,* but our *measure* of probability comes from past observations. The more information we have about past events, the better is our prediction of a future possibility.

Recently, a colleague, D. L. Wiegman, brought me some interesting data which involved one interpretation of probability. Essentially, he had used a well-defined protocol to hemorrhage rats for 1 hour. He had records of body weight (grams) at the time of hemorrhage and death (d) or survival (s) for the 24-hour period which followed reinfusion of the hemorrhaged volume. These data were: 166/s, 196/d, 146/d, 191/s, 204/s, 153/d, 208/d, 206/s, 220/d, 119/d, 185/s, 175/s, 151/d, 140/s, 140/d, 139/d, 179/s, 158/d, 161/s, 143/s, 155/d, 144/d, 145/s, 180/s, 196/s, 172/s, 172/s, 112/d, 159/s, 120/d, 115/d, 116/s, 127/d, 155/d, 117/s, 110/d, 114/s, 150/s. The average weight was 156 gm, with a range from 110 to 220 gm and survival was 52.6 percent (20sX100/38). At this point, one might predict that any future rat would have a 52.6 percent probability of being alive 24 hours after hemorrhage with the protocol. Is this a good prediction regardless of the weight of this future rat?

An investigator might attempt to answer this question by construction of a histogram[1] for the existing data. One such attempt would give a histogram with "percent survival in each weight class" on the

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ordinate and with "body weight in grams" on the abscissa. The abscissa would be divided into 8 class intervals with a starting point of 100 gm and with an interval size of 15 gm. This histogram would be difficult to interpret, however, because 6 of the intervals would contain 5 or less observations (animals).

For the individual weight/survival data above, we posed the question, "Is the probability for survival independent of body weight (ie, the same for weights between 110 and 220 gm)?" We will approach the answer to this question by use of a *cumulative distribution* curve (fig 1). For our data, the abscissa is expressed as "body weight in grams" with "steps" which are *as small as* possible. Ideally, we should have *one* observation for each abscissa step from the smallest weight to the largest weight, so we have selected a 3-gm step size (approximately [200-110]/38). The first step is selected by the formula: the "largest weight" (220 gm) minus the product of the "step size" (3) and the "number of observations less one" $(38 - 1)$. Thus, our starting point is $220 - (3 \times 37)$ or 109 gm. For each "step" on the abscissa, an ordinate value is calculated as the cumulative survival for *all* observations with body weight *less than* or *equal to* $(\leq)$ the "step weight."

For example, the 1st step is 109 gm. Since our data do not contain any animals with weight $\leq$ 109 gm, we move to the next step $(109 + 3 = 112$ gm). Our data contain 2 animals (112/d and 110/d) with weights $\leq$112 gm. Since neither of these animals survived, the cumulative survival (ordinate value) is 0 percent. The next step $(112 + 3 = 115)$ contains 4 animals (112/d, 115/d, 110/d, 114/s) with weights $\leq$115 gm. The cumulative survival is 25 percent. The cumulative survival is calculated for each of the remaining steps in a similar fashion and is plotted on the ordinate in figure 1. The last step $(217 + 3 = 220)$ includes all animals, and thus has the ordinate value which is the survival (52.6 percent) for all observations.

How does one interpret this cumulative distribution curve to investigate the possibility of a relationship between survival and body weight? Perhaps an insight for this interpretation can be gained from examination of theoretic data. I have generated a sequence of 38 theoretic observations of survival and death by using a table of random numbers to assign 1 observation to each 3-gm step over the range 109 to 220 gm.

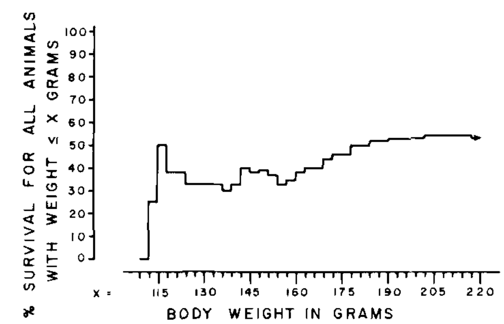

FIG 1. Cumulative distribution curve for survival of 38 rats during the 24-hour period following hemorrhage. (Data from D. L. Wiegman)

Each observation has a 50 percent chance of being a death or a survival and there is no relationship between survival probability and "weight."

A cumulative distribution curve for this theoretic data is shown (fig 2); this curve demonstrates the characteristic of "convergence," in that the percent survival for the first 10 to 16 steps is above *or* below the "real" probability of 50 percent; however, the "error" between the calculated value and the "real" value becomes smaller as more steps are added.

This property of convergence is better demonstrated in another manner. A cumulative distribution curve is arbitrarily constructed for the lower *one-half* (19) of the total number of theoretic observations. This curve appears in the left half of figure 3 for steps from 109 to 163 gm. Another cumulative distribution curve is constructed for the upper one-half (19) of the total number of observations by *starting with the highest* weight and stepping downward (right half of fig 3). In this curve, the ordinate value for each step is the cumulative survival for

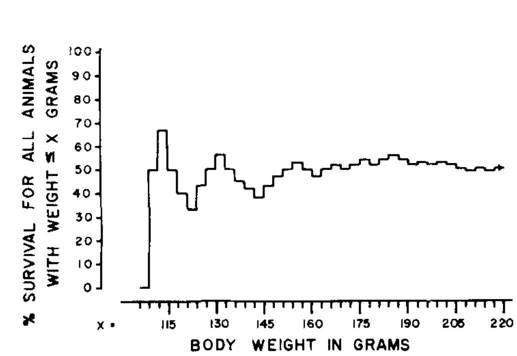

FIG 2. Cumulative distribution curve for survival of 38 "theoretic" animals for which the probability of survival is 50 percent and is independent of body weight. The sequence of survival and death observations was obtained from a random number table.[2]

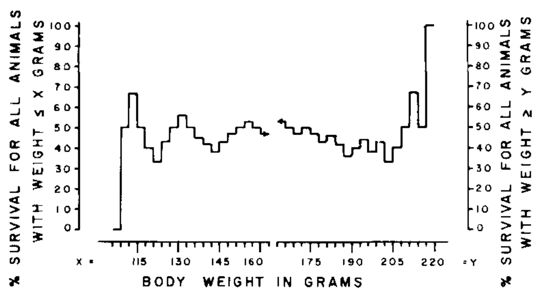

FIG 3. Cumulative distribution curves for survival of 19 "theoretic" animals with weights ≤163 gm (left panel) and 19 "theoretic" animals with weights ≥163 gm (right panel). The sequence of theoretic survival and death observations is the same as used for the cumulative distribution curve in fig 2.

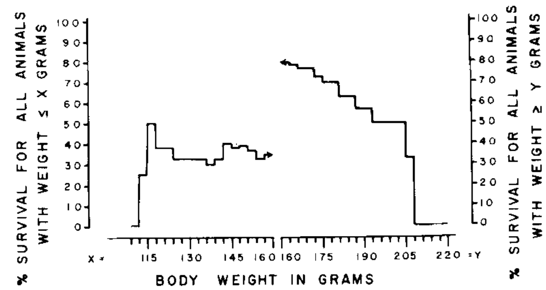

FIG 4. Cumulative distribution curves for survival of 23 rats with weights ≤160 gm (left panel) and for 15 rats with weights ≥160 gm (right panel) during the 24-hour period following hemorrhage. These were obtained from the same raw data as used for the cumulative distribution curve in fig 1.

all observations (theoretic animals) with weight *greater than* or *equal* to (≥) the abscissa value. (For example, the first step is 220 − 3 = 217 and the ordinate is percent survival for all "animals" with weight ≥217 gm.) Figure 3 demonstrates convergence because the survival (47.3%) in the last step (160-163 gm) for the lower cumulative curve (left panel) approaches the survival (52.6% in the last step (166-163 gm) for the upper cumulative curve (right panel). These survivals (47.3% and 52.6%) also approach (ie, converge toward) the "real value" of 50 percent.

Comparison of our animal data (fig 1) to our theoretic data (fig 2) suggests the absence of convergence in the animal data (fig 1). The cumulative distribution in figure 1 appears to converge toward 35 percent until step 163 is reached. Following this step, the cumulative curve appears to converge toward the survival of 52.6 percent for the "last step" (217-220). This suspicion of *divergence* (lack of convergence) is better investigated by arbitrarily dividing the data at the point which precedes the suspected change (step 160).

Figure 4 shows the cumulative distribution curve for 23 animals with weight ≤160 gm (left panel) and that for 15 animals with weight ≥160 gm (right panel). In contrast to "convergence" for our theoretic data in figure 3, our animal data in figure 4 appear to *diverge* (35% survival for the 157-160 step in the left curve and 80% survival for the 163-160 step in the right curve). This suggests that body weight in the 110 to 220 gm range influences survival of hemorrhage. In other words, we would *not predict* that a future rat would have a 52.6 percent probability of being alive 24 hours after hemorrhage, with my colleague's protocol.

As a conclusion to this discussion on probability and cumulative distribution curves, several points need emphasis:

(a) The end (last) step in a cumulative distribution gives a "best estimate" of probability; yet, it does not necessarily give the exact probability (or chance) that an event will happen.

(b) The "lower" and "upper" cumulative distributions can *indicate* convergence if the end step probabilities are as close to each other as in figure 3, or divergence if they are as far from each other as in figure 4. (How close is "close?" This is most appropriately answered by *statistical* tests, which have not been discussed here.)

(c) Cumulative distribution curves usually work very well for 18 or more observations, whereas histograms usually require 40 or more observations to give useful interpretations.

(d) Cumulative distribution curves *do not prove* or statistically test anything; yet, they can lead to a hypothesis which could otherwise be overlooked and which can subsequently be tested statistically.

REFERENCES

1. Harris PD: Statistical Pearls. II. How often did that happen? Anesth Analg 55:357-359, 1976

2. Dixon WJ, Massey FJ: Introduction to Statistical Analysis. New York, McGraw-Hill, 1957